

Design of bi-power reaching law sliding mode controller based on exponential observer

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Abstract—Considering the complex nonlinear characteristics of the quadrotor system, such as underactuation, strong coupling and susceptibility to disturbance, the disturbance in real time can be estimated by the exponential convergent disturbance observer. Using the disturbance is input compensation. The exponentially convergent disturbance observer is used to resist unknown disturbances. In this paper, a double power reaching law sliding mode controller based on the exponential observer (EO-DSMC) is proposed by combining the exponential observer and the double power reaching rate sliding mode control method, it is applied to the attitude subsystem of the quadrotor unmanned aerial vehicle (UAV). Lyapunov theory is used to prove the stability of the system. Comparisons of the simulation of the single-machine flight between before and after adding the controller revealed that the designed controller is used to control UAV attitude better and enhanced system stability and anti-interference.

Keywords—Exponential observer, Double power reaching law, Attitude subsystem, Lyapunov theory

I. INTRODUCTION

In recent years, with the rapid emergence of UAVs, findings have greatly increased. UAV do not need to consider the physical limits of their flight. More intense maneuvers can be used for UAV. However, as maneuvering movement is often accompanied by large angle changes[1], Nonlinearity and coupling are considered as the dynamic characteristics of UAV. The characteristics of UAV are of great significance to the research of UAV. A sliding mode control method based on double closed-loop control strategy is proposed to deal with the trajectory tracking problem of quadrotor UAV affected by external disturbance in [2]. Disturbance observer and non-singular terminal sliding mode control are applied to the inner ring, integral terminal sliding mode method is used to the outer ring. The attitude stability and tracking accuracy of UAV are achieved by the inner and outer rings. In [3], an improved adaptive sliding mode control is designed to solve the problem of uncertainty and

external interference in UAV navigation control. The problems of parameter uncertainty and external disturbance robustness are solved by the proposed controller. The trajectory tracking is also maintained stably due to the controller. In order to realize the trajectory tracking of UAV more effectively, [4] showed that the steady state control of quadrotor UAV trajectory tracking can be achieved by a backstep method and dual-loop cascade control method.

In view of the complex nonlinear characteristics of the quadrotor system, such as underactuation, strong coupling and susceptibility to disturbance, the unknown disturbance is regarded as a total disturbance. The controller design is based on the traditional sliding mode control^[5], the chattering problem of the sliding mode needs to be considered. The controller designed in this paper has not been reported yet. This paper puts forward the attitude subsystem controller applied to the attitude subsystem of quadrotor UAV for the attitude subsystem with unknown interference. A two-power reaching law sliding mode controller based on the exponential observer (EO-DSMC) was designed by combining the exponential observer and the double-power reaching rate sliding mode control method[6]. The given expected attitude angles are input into the designed controller (EO-DSMC), the interference signals observed by exponential observer are put in attitude subsystem for compensation adjustment. Then, the new attitude angle signals are obtained in attitude subsystem and put into the designed controller. Finally, the closed-loop control is formed and the real-time update is implemented. In this controller, the exponential observer is used to solve the unknown disturbance of the attitude subsystem, the double power reaching rate is used to reduce chattering.

There are four sections in this paper. In Section II, this section gives a brief overview of the four-rotor model with disturbance. In Section III, the analysis will highlight the design of double power reaching rate sliding rate sliding mode controller based on exponential observer. In Section IV, the parts will analyze the results of simulations. Finally, Section V is devoted to the final conclusions.

II. ESTABLISHMENT OF A FOUR-ROTOR MODEL WITH DISTURBANCE

Quadrotor UAV is a kind of rotor UAV, which can be used for hovering and vertical takeoff and landing. Small size, low price and easy operation are all regarded as the characteristics of the quadrotor UAV. Multi-input, multi-output, strong coupling and underdrive are considered to be the characteristics of the dynamics model of the quadrotor UAV^{[7][8]}. For the complex nonlinear characteristics of the quadrotor system, exponential convergent disturbance observer is considered to estimate the disturbance in real time. at the same time, disturbance is regarded as input compensation. So the exponential observer is considered to have the ability to resist unknown disturbance. The unknown disturbance of the attitude subsystem is added to the attitude subsystem as total disturbance $d_i (i=1,2,3)$. After adding the total disturbance, the attitude subsystem is.

$$\begin{cases} \ddot{\theta} = u_2 - lK_4\dot{\theta} / I_x + d_1 \\ \ddot{\psi} = u_3 - lK_5\dot{\psi} / I_y + d_2 \\ \ddot{\phi} = u_4 - lK_6\dot{\phi} / I_z + d_3 \end{cases} \quad (1)$$

Where, θ , ψ and ϕ respectively represent the roll angle, pitch angle and yaw angle of the quadrotor UAV; $\dot{\theta}$, $\dot{\psi}$ and $\dot{\phi}$ are the first derivative; $\ddot{\theta}$, $\ddot{\psi}$ and $\ddot{\phi}$ are the second derivative; u_2 , u_3 , u_4 represent the virtual control input; $K_i (i=4,5,6)$ represents the resistance coefficient; l is the distance between the rotor center and the geometric center of the body; I_x , I_y , I_z respectively represent the moment of inertia of the three axes.

For the position subsystem, the traditional sliding mode is used to design the sliding mode surface. As a case in point, is taken the sliding mode function $s_1 = c_1 x_e + \dot{x}_e$, where $c_1 > 0$, $x_e = x - x_d$. Exponential reaching rate $\dot{s} = -\varepsilon_1 s - \eta_1 \text{sgn}(s)$, $\varepsilon_1 > 0$, $\eta_1 > 0$ is selected, the sliding mode control law is designed as $u_{1x} = -c_1 \dot{x}_e + K_1 \dot{x} / m + \ddot{x}_d - \varepsilon_1 s_1 - \eta_1 \text{sgn}(s_1)$. In order to further weaken chattering caused by sliding mode control, hyperbolic tangent function $\tanh(\cdot)$ is used instead of sign function $\text{sgn}(\cdot)$.

The control law design of position subsystem u_{1y} and u_{1z} is similar to u_{1x} , so the final position subsystem control law is designed as

$$\begin{aligned} u_{1x} &= -c_1 \dot{x}_e + K_1 \dot{x} / m + \ddot{x}_d - \varepsilon_1 s_1 - \eta_1 \tanh(s_1), \\ u_{1y} &= -c_2 \dot{y}_e + K_2 \dot{y} / m + \ddot{y}_d - \varepsilon_2 s_2 - \eta_2 \tanh(s_2), \\ u_{1z} &= -c_3 \dot{z}_e + K_3 \dot{z} / m + \ddot{z}_d - \varepsilon_3 s_3 - \eta_3 \tanh(s_3). \end{aligned} \quad (2)$$

III. DESIGN OF DOUBLE POWER REACHING RATE SLIDING RATE SLIDING MODE CONTROLLER BASED ON EXPONENTIAL OBSERVER

The unknown disturbances are regarded as the disturbances that can be observed by the exponential converging disturbance observer^[9]. Then, the total disturbance is designed to be observed by the exponential observer, the disturbances observed by the observer are compensated in the sliding mode control. Due to the chattering defects of the sliding mode control, the chattering is reduced by considering the double-power reaching law. When designing the sliding mode controller, the double power reaching law sliding mode controller based on exponential observer is designed by combining observer interference and double power reaching rate.

Taking pitch angle θ as an example, the designed exponential observer is:

$$\begin{cases} \dot{\mu} = n(lK_4\dot{\theta} / I_x - u_2) - n\hat{d}_1 \\ \hat{d}_1 = \mu + n\theta \end{cases} \quad (3)$$

Where, μ is the auxiliary parameter vector defined in the observer; $\dot{\mu}$ is the first derivative, which should meet the following requirements: $\dot{\mu} = n(lK_4\dot{\theta} / I_x - u_2) - n(\mu + n\theta)$; n is the coefficient of the auxiliary parameter vector; \hat{d}_1 is the interference estimate corresponding to the roll angle.

The disturbance observed by the above observer is compensated in the sliding mode control, the sliding mode surface is designed as $s = c_1 \theta_e + \dot{\theta}_e$, and $c_1 > 0$; the roll angle error is θ_e , and $\theta_e = \theta_d - \theta$, the first derivative is $\dot{\theta}_e$, θ_d indicates the expected roll angle value.

Because of the chattering defect of sliding mode control, the double power reaching rate is considered in the design of EO-DSMC controller. The double power approaching law can be expressed as $\dot{s} = -\alpha |s|^{k_1} \text{sgn}(s) - \beta |s|^{k_2} \text{sgn}(s)$.

α and β are the reaching rate coefficients respectively; k_1 and k_2 are the double power index parameters of the reaching rate respectively, which need to satisfy $\alpha, \beta > 0$, $k_1 > 0$, $0 < k_2 < 1$; the sign function is $\text{sgn}(s)$.

Therefore, design of sliding mode controller with double power reaching rate based on exponential observer the pitch angle control rate of four rotor UAV is

$$\begin{aligned} u_2 &= c_4 \dot{\theta}_e + lK_4 \dot{\theta} / I_x + \ddot{\theta}_d - \hat{d}_1 \\ &+ \alpha_1 |s_4|^{k_1} \text{sgn}(s_4) + \beta_1 |s_4|^{k_2} \text{sgn}(s_4). \end{aligned} \quad (4)$$

Where, c_4 is the sliding surface parameter, and $c_4 > 0$;

\hat{d}_1 is the estimated value of interference corresponding to the pitch angle; s is the sliding mode surface function; \dot{s} is the first derivative of the sliding mode surface function; α_1 , β_1 respectively are the approaching rate coefficient of the pitch angle, k_1 , k_2 respectively are the double power index parameters of the approaching rate of the pitch angle, which should satisfy $\alpha_1 > 0$, $\beta_1 > 0$, $k_1 > 0$, $0 < k_2 < 1$.

The stability of attitude controller (EO-DSMC) proposed is analyzed in this paper. the Lyapunov function is selected: $V = (s^2 + \tilde{d}_1^2)/2$. Take the first derivative of the function: $\dot{V} = s\dot{s} + \tilde{d}_1\dot{\tilde{d}}_1 \leq 0$. Among them, $\alpha \geq |d_1|_{\max}$, $\beta \geq |d_1|_{\max}$. The initial error of the observer is $\tilde{d}_1(0)$, assuming $|\tilde{d}_1(0)| = |\tilde{d}_1|_{\max}$, the observer error is also based on Lyapunov stability. Therefore, the real-time estimation of the total interference d_1 can be well realized by the observed value \hat{d}_1 .

Then the convergence analysis of the attitude subsystem control input is obtained $\dot{V} \leq 0$, convergence and system stability can be realized in the attitude subsystem.

Similarly, the control input u_3 , u_4 can be obtained:

$$\begin{aligned} u_3 &= c_5 \dot{\psi}_e + lK_5 \dot{\psi} / I_y + \ddot{\psi}_d - d_2 \\ &\quad + \alpha_2 |s_5|^{k_3} \text{sgn}(s_5) + \beta_2 |s_5|^{k_4} \text{sgn}(s_5), \\ u_4 &= c_6 \dot{\phi}_e + lK_6 \dot{\phi} / I_z + \ddot{\phi}_d - d_3 \\ &\quad + \alpha_3 |s_6|^{k_5} \text{sgn}(s_6) + \beta_3 |s_6|^{k_6} \text{sgn}(s_6). \end{aligned} \quad (5)$$

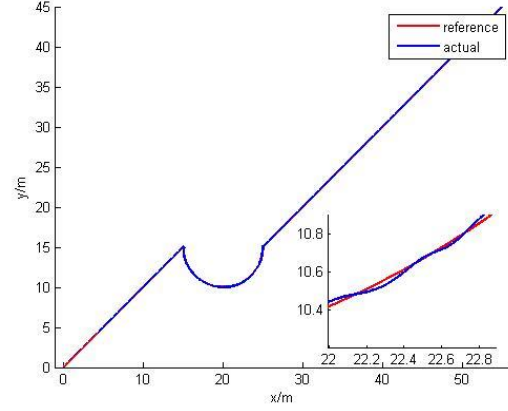
Where, $c_5 > 0$, $\alpha_2 > 0$, $\beta_2 > 0$; $k_3 > 0$, $0 < k_4 < 1$; $c_6 > 0$, $\alpha_3 > 0$, $\beta_3 > 0$, $k_5 > 0$, $0 < k_6 < 1$. The settings of u_3 , u_4 are similar to those of u_2 and are not detailed here.

IV. SIMULATION RESULTS AND ANALYSIS

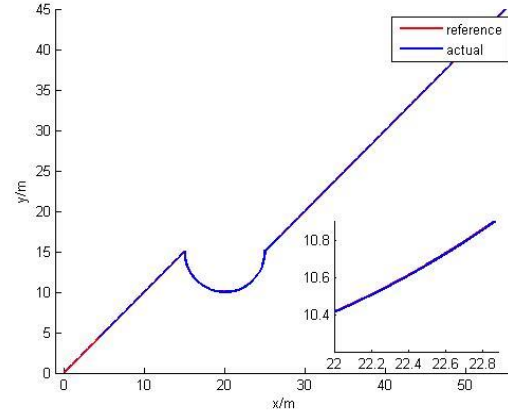
In order to verify the proposed controller, a quadrotor aircraft was selected for flight simulation. All quadrotor models in the simulation were the same as formula (1). the parameters of quadrotor UAV controlle is: $c_1 = c_2 = c_3 = 1$, $c_4 = c_5 = c_6 = 5$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 2$, $\eta_1 = \eta_2 = \eta_3 = 0.01$, $\alpha_1 = \alpha_2 = \alpha_3 = 2$, $\beta_1 = \beta_2 = \beta_3 = 2$, $k_1 = k_3 = k_5 = 2$, $k_2 = k_4 = k_6 = 0.2$. The initial position of the single machine was set as $(x, y, z) = (0, 0, 0)$, the initial angle was $(\theta, \psi, \phi) = (0, 0, 0)$. Setting exponential convergence disturbance observer parameter was $n = 100$, the expected

yaw angle was set as $\psi = 0 \text{ rad}$, the desired trajectory and perturbation were set. Where, the disturbance was set as $d_1 = 0.5 \sin(t)$, $d_2 = 0.5 \sin(t)$, $d_3 = 0.5 \sin(2t)$.

The simulation verification of single machine is carried out in the two cases of disturbance without exponential convergence observer and disturbance with exponential convergence observer. In order to verify the validity of the designed controller, the traditional sliding mode controller is used in the position loop, the double power reaching law sliding mode controller based on the exponential observer is applied to the attitude loop to suppress interference.



(a) Interference with no observer



(b) There's disturbance, there's observer

Fig.1 Two dimensional images of single plane flight in two cases

In the case of interference, Fig. 1 shows Two dimensional images of single plane flight in two cases. Fig. 1(a) shows the single-machine flight 2D images obtained without adding the index observer designed in this paper, Fig. 1(b) shows the single-machine flight 2D images obtained by adding the controller designed in this paper. The comparison of 3D track images in the above two cases is not obvious. After local amplification of the 2D image, it can be seen that the image fluctuates in the case of interference and no observer. Next, the position and attitude tracking errors with disturbance and observer are analyzed.

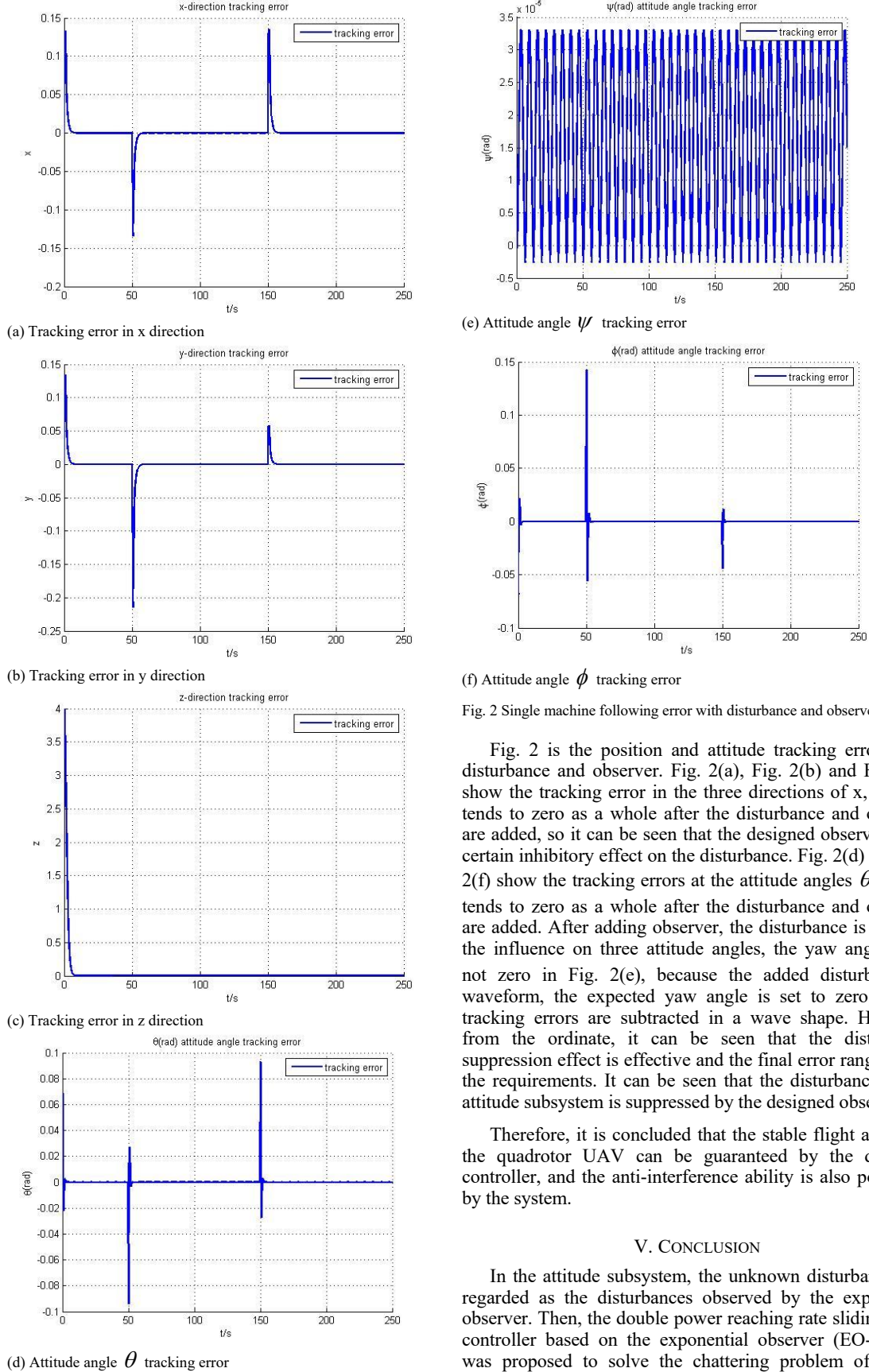


Fig. 2 Single machine following error with disturbance and observer

Fig. 2 is the position and attitude tracking errors with disturbance and observer. Fig. 2(a), Fig. 2(b) and Fig. 2(c) show the tracking error in the three directions of x, y and z tends to zero as a whole after the disturbance and observer are added, so it can be seen that the designed observer has a certain inhibitory effect on the disturbance. Fig. 2(d) and Fig. 2(f) show the tracking errors at the attitude angles θ and ϕ tends to zero as a whole after the disturbance and observer are added. After adding observer, the disturbance is reduced the influence on three attitude angles, the yaw angle ψ is not zero in Fig. 2(e), because the added disturbance is waveform, the expected yaw angle is set to zero, so the tracking errors are subtracted in a wave shape. However, from the ordinate, it can be seen that the disturbance suppression effect is effective and the final error range meets the requirements. It can be seen that the disturbance of the attitude subsystem is suppressed by the designed observer.

Therefore, it is concluded that the stable flight ability of the quadrotor UAV can be guaranteed by the designed controller, and the anti-interference ability is also possessed by the system.

V. CONCLUSION

In the attitude subsystem, the unknown disturbances are regarded as the disturbances observed by the exponential observer. Then, the double power reaching rate sliding mode controller based on the exponential observer (EO-DSMC) was proposed to solve the chattering problem of sliding

mode control. The observed disturbances were fed back into the control law to compensate, the simulation experiment of UAV flight was carried out. The simulation results of the controller designed in this paper are as follows:

(1) From the UAV flight track, the stability of single aircraft flight is improved by the controller designed.

(2) With position and attitude tracking errors, interference can be observed by the designed controller, the anti-interference ability is possessed by the system.

ACKNOWLEDGEMENT

The work is supported by the Key R & D plan of Shaanxi Province(2020ZDLGY06-01) and Science & Technology Innovation Guidance Project of Xi'an, China (21JY033). Science & Technology Innovation Guidance Project of Xi'an, China (No. 2020KJRC0087).

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